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SYSTEM OPTIMIZATION BY PERIODIC CONTROL.(U)  
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AFOSR-TR-78-0012

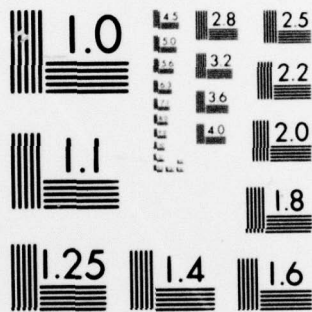
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Interim Scientific Report

for

United States Air Force Grant No. AFOSR 77-3158

*per book  
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SYSTEM OPTIMIZATION BY PERIODIC CONTROL

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Period: October 1, 1976 to October 1, 1977

## Summary of Research Effort

The work undertaken during the period October 1, 1976 to October 1, 1977 was in two main areas: the theory and computation of periodic controls, and the input-output characterization of nonlinear systems. The second area impinges on the first since it offers new approaches to analyzing the effects of periodic forcing in nonlinear systems. The Bibliography gives a chronological listing of papers prepared during the year. Items (1, 5, 6, 7) were revised from earlier work for conference presentation or journal publication; items (2, 3, 4, 8) represent entirely new research. In what follows a brief description of research which was carried out will be given. It includes activities which go beyond (2, 3, 4, 8) and will be continued into the coming year.

The effort on periodic control was devoted almost entirely to: sufficient conditions for optimality, a computational study of periodic aircraft cruise, the use of variational models for analyzing the effect of system nonlinearities in the presence of periodic forcing. Item (3) extends the research in (1) and (6) by examining certain sufficient conditions for optimality in a general class of periodic control problems. These conditions give some indication of circumstances under which periodic control cannot improve performance. For example, improvement is impossible if the plant is modeled by linear differential equations and the cost functional is convex. This underlines the importance of understanding the effects of nonlinearities in periodic control problems. An interesting application of periodic control is aircraft cruise (Gilbert and Parsons, Journ. Aircraft, Vol 13, pp. 828-

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830, 1976). An accurate fourth-order model of subsonic cruise was developed in non-dimensional form. Approximate analytical techniques, which were applied to the model, showed that periodic cruise might produce better performance in the presence of an altitude constraint. This led to the development of an algorithm for obtaining optimal periodic cruise trajectories. Preliminary computations have shown that in some cases (low drag, low wing loading) significant improvements in cruise performance are possible. The difficulty of the computations has shed light on issues concerning the development of general algorithms for solving periodic control problems. These issues will be investigated in the coming year. Also, when additional computations are finished, a report describing the results of the aircraft studies will be prepared. The Volterra transfer function offers an approach to evaluating the response of nonlinear systems with periodic forcing. Investigations of applications to periodic control was begun. It appears that tools of the type described in (7) are of value.

The investigation of input-output characterizations for nonlinear systems also has applications to the realization problem for nonlinear systems. Previous efforts on such problems have focused on algebraic techniques, and the research reported in (2,4,8), which is based on variational series, represents a departure. Necessary and sufficient conditions for the finite dimensional realization of bilinear and 2-power input-output maps are considered in (4,8). The problem of obtaining minimal order realizations for nonlinear systems is difficult and has not been solved previously. For the special case of single-input, single-output, continuous-

time 2-powers, (2) gives the answer in a simple, useful form. These research directions were explored further with the help of A. E. Frazho who was a post doctoral researcher working on the grant during the summer months. Starting with ideas similar to those in (7, 8) the backward shift realization, used previously in the study of infinite-dimensional linear systems, was applied to multivariable 2-powers. Interesting results were obtained in both the infinite-dimensional and finite-dimensional cases. In fact, it appears that the backward shift approach offers notable advantages in the development of new algorithms for minimal realizations of both linear and polynomial nonlinear systems. This line of research is being continued and should lead to several reports in the coming year.



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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

1. REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM	
1. REPORT NUMBER <b>AFOSR-TR-78-8812</b>	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) <b>SYSTEM OPTIMIZATION BY PERIODIC CONTROL.</b>		5. TYPE OF REPORT & PERIOD COVERED <b>Interim scientific rept.</b>	
6. AUTHOR(s) <b>Elmer G. Gilbert</b>		7. PERFORMING ORGANIZATION NAME AND ADDRESS <b>The University of Michigan Department of Aerospace Engineering Ann Arbor, MI 48109</b>	
8. CONTROLLING OFFICE NAME AND ADDRESS <b>Air Force Office of Scientific Research/NM Bolling AFB, DC 20332</b>		9. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS <b>61102F 2304 A1</b>	
10. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		11. REPORT DATE <b>11/1977</b>	
		12. NUMBER OF PAGES <b>5</b>	
		13. SECURITY CLASS. (of this report) <b>UNCLASSIFIED</b>	
14. DISTRIBUTION STATEMENT (of this Report) <b>Approved for public release; distribution unlimited.</b>		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)			
17. SUPPLEMENTARY NOTES			
18. KEY WORDS (Continue on reverse side if necessary and identify by block number)			
19. ABSTRACT (Continue on reverse side if necessary and identify by block number) Research was conducted in two areas: the theory and computation of periodic controls, and the input-output characterization of nonlinear systems. The effort on periodic control was devoted almost entirely to: sufficient conditions for optimality, a computational study of periodic aircraft cruise, the use of variational models for analyzing the effect of system nonlinearities in the presence of periodic forcing. The investigation of input-output characterizations for nonlinear systems was extended to the realization problem for nonlinear systems.			